Chapter 29
Space Coverage Analysis of BeiDou Navigation Satellite System Regional Constellation

Zhenhai Li, Wenhai Jiao, Jin Fan, Changjiang Geng and Yu Bai

Abstract Satellites constellation space coverage analysis plays an important reference role in GNSS spatial positioning, navigation and timing, guides the designing of future constellation. This paper used regional constellation of BeiDou navigation satellite system composed of 5GEO, 5IGSO and 4MEO satellites, based on geometric relationship between satellite, earth and focused point, computed worldwide spatial visible satellites number and PDOP, analyzed the statistical results, and showed the spatio-temporal distribution. Computational results showed that: BeiDou regional navigation satellite system could provide effective PNT services for Asia–Pacific region and spatial south and north Polar Regions under 1,000 km, satisfied the designed requirements, but not the space above 1,000 km because of the promptly reducing of spatial coverage range, increasing of effective continual time segments, decreasing of maximum continual time and total time in all segments.

Keywords BeiDou · Space coverage · PNT services · Visible satellites number · PDOP

29.1 Introduction

BeiDou is a global navigation satellite system which is self-dependent developed and operated by China, and it is one of four main GNSS in the world. BeiDou will provide positioning, navigation and timing (PNT) services for global users by

Z. Li (✉) · W. Jiao · J. Fan · C. Geng · Y. Bai
Test and Assessment Research Center, China Satellite Navigation Office,
Beijing 100094, China
e-mail: lizhenhai666@163.com

W. Jiao
Beijing Institute of Tracking and Telecommunication Technology,
Beijing 100094, China

J. Sun et al. (eds.), *China Satellite Navigation Conference (CSNC) 2013 Proceedings*, Lecture Notes in Electrical Engineering 243,
DOI: 10.1007/978-3-642-37398-5_29,

2020. According to the development plan [1, 2], the working satellites of BeiDou include 5GEO (Geosynchronous orbit) satellites, 5IGSO (Inclined Geosynchronous Orbits) satellites and 4MEO (Medium Earth Orbit) satellites in December 2012, and will completely provide stable and reliable PNT services for the Asia–Pacific region by early 2013.

Space coverage analysis is one of the important characteristics for GNSS constellation performance evaluation, which plays an important role in the world-wide terrestrial and spatial PNT services and guides the future constellation design of BeiDou system. Jiejuan et al. [3] established the geometrical models between GPS satellites and LEO, MEO and HEO satellite respectively, and simulated the coverage performance of 32 GPS satellites constellation. Zhang et al. [4] analyzed the constellation's observation region, summarized the main influential factors on space coverage performance, illuminated the judgment approaches, lastly analyzed the coverage performance in spatio-temporal domain based on theoretical constellation. Besides, GPS space service volume and spatial navigation experiments [5–7] were deeply researched in foreign countries.

With the development of spatial technologies, spatial navigation is increasingly concerned in the satellite navigation fields at present. The GPS navigation capabilities for LEO even HEO are focused by more and more researchers, and achieved engineering application gradually. Because BeiDou was developed and started to provide Initial Operational Capability (IOC) lately, its spatial navigation capabilities are less concerned. This paper mainly discusses the world-wide spatial coverage performance based on the regional constellation of BeiDou navigation satellite system, firstly develops several computational models of spatial visible satellites number and DOPs, then computes these values in world-wide spatial domain, and analyzes the spatial distribution and characters of these factors lastly.

29.2 Basic Principles

Satellite visibility and DOPs are two key factors in GNSS service performance evaluation. The former one denotes the number of satellites observed by a receiver, represents the PNT services capabilities that GNSS can provide for special spatio-temporal users, and the later one denotes the scale coefficient between pseudo-range errors and user positioning accuracy, that is varied with the spatial geometrical distribution between satellites and receiver [8]. Both visible satellites number and DOPs can be expressed as functions of time and space, that is to say, two factors will vary with the changing of spatio-temporal domain.

29.2.1 Spatial Satellites Visibility

In order to conveniently analyze the visibility of satellite S on a spatial focused point F, the space above the earth surface is divided into three independent research spaces, including the LEO space (Space I) from earth surface to h_1 which is the altitude of a tangency point of satellite's conical signal to earth's concentric circles, the MEO space (Space II) from h_1 to h_0 which is the orbit altitude of navigation satellite S, and the HEO space (Space III) beyond h_0. These three areas are shown from Figs. 29.1, 29.2 and 29.3, O is the earth geocentre, θ denotes the angle between SO which is a line from satellite to earth center and tangency line which is satellite to the earth surface, φ denotes half beam width angle of satellite's transmitting signal, α and β are two internal angles corresponding to two vertexes F and S in ΔSFO, h_0 and h_1 denotes the boundary altitude of three independent spaces. Assuming the coordinates of the spatial focused point F and satellite S are (x_F, y_F, z_F) and (x_S, y_S, z_S) under ECEF respectively, then the distances R_S between satellite and earth geocentre, R_F between focused point F and earth geocentre, R_{SF} between satellite S and focused point F can be expressed as

$$R_S = \left(x_S^2 + y_S^2 + z_S^2\right)^{1/2}$$

$$R_F = \left(x_F^2 + y_F^2 + z_F^2\right)^{1/2}$$

$$R_{SF} = \left[(x_S - x_F)^2 + (y_S - y_F)^2 + (z_S - z_F)^2\right]^{1/2} \tag{29.1}$$

Internal angles α and β can be described as

$$\alpha = \arccos\left(\frac{R_{SF}^2 + R_F^2 - R_S^2}{2R_{SF}R_F}\right)$$

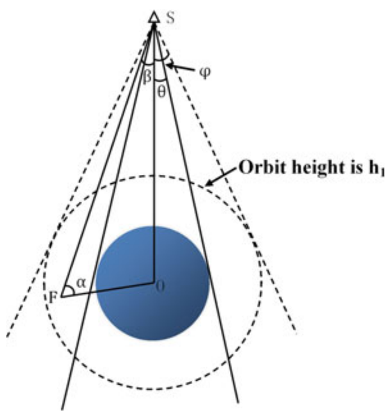

Fig. 29.1 Object point F in space I

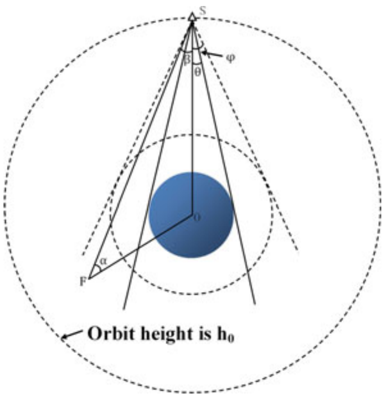

Fig. 29.2 Object point F in space II

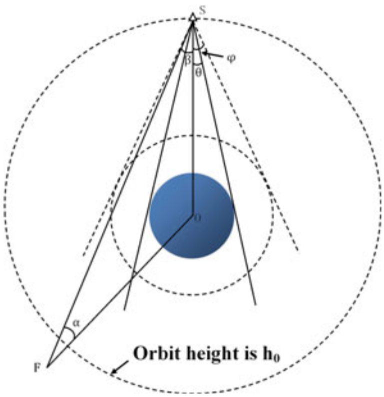

Fig. 29.3 Object point F in space III

$$\beta = \arcsin\left(\frac{R_F}{R_S}\sin\alpha\right) \tag{29.2}$$

Satellite S's visibility on spatial focused point F can be derived from α and β, the visibility on the focused point F in Space I ($0 \le h < h_1$, Fig. 29.1) is defined as

$$\text{Vis} = \begin{cases} \text{True}, & \alpha \le 90^\circ \text{ and } \theta \le \beta \le \varphi \\ \text{True}, & \alpha > 90^\circ \textit{ and } \theta \le \beta < \varphi \\ \text{True}, & \alpha > 90^\circ \textit{ and } 0^\circ \le \beta < \theta \\ \text{False}, & \alpha \le 90^\circ \text{ and } 0^\circ \le \beta < \theta \end{cases} \tag{29.3}$$

The visibility on the focused point F in Space II ($h_1 \le h < h_0$, Fig. 29.2) is

$$\text{Vis} = \begin{cases} \text{False}, & \alpha \le 90^\circ \text{ and } \beta > \varphi \\ \text{True}, & \alpha \le 90^\circ \text{ and } \theta \le \beta < \varphi \\ \text{False}, & \alpha \le 90^\circ \text{ and } 0^\circ \le \beta < \theta \\ \text{False}, & \alpha > 90^\circ \; and \; \beta \ge \varphi \\ \text{True}, & \alpha > 90^\circ \; and \; \theta \le \beta < \varphi \\ \text{True}, & \alpha > 90^\circ \; and \; 0^\circ \le \beta < \theta \end{cases} \tag{29.4}$$

The visibility on the focused point F in Space III ($h_0 \le h$, Fig. 29.3) is

$$\text{Vis} = \begin{cases} \text{False}, & \alpha < 90^\circ \; and \; \beta \ge \varphi \\ \text{True}, & \alpha < 90^\circ \; and \; \theta \le \beta < \varphi \\ \text{False}, & \alpha < 90^\circ \; and \; 0^\circ \le \beta < \theta \end{cases} \tag{29.5}$$

Without consideration of spatial point's masking angle, satellite S's visibility on spatial focused point F can be concluded through formula (29.3)–(29.5), then the number of visible satellites of constellation can be calculated.

29.2.2 DOP Computation

Dilution of precision (DOP) is a term used in GNSS to specify the additional multiplicative effect of GNSS satellite geometry on GNSS precision. GDOP, HDOP, VDOP, PDOP and TDOP are Geometric, Horizontal, Vertical, Positional, and Time dilution of precision respectively, and can be computed by observation equations coefficient matrix H and coordinate system translation matrix T.

The coefficient matrix H of pseudo-range observation equations in GNSS absolute positioning can be described as

$$H = \begin{bmatrix} \frac{X_1^s - X_0}{\rho_1} & \frac{Y_1^s - Y_0}{\rho_1} & \frac{Z_1^s - Z_0}{\rho_1} & -1 \\ \frac{X_2^s - X_0}{\rho_2} & \frac{Y_2^s - Y_0}{\rho_2} & \frac{Z_2^s - Z_0}{\rho_2} & -1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{X_n^s - X_0}{\rho_n} & \frac{Y_n^s - Y_0}{\rho_n} & \frac{Z_n^s - Z_0}{\rho_n} & -1 \end{bmatrix} \tag{29.6}$$

where $\left(X_n^s, Y_n^s, Z_n^s\right)$ is the nth satellite's position, (X_0, Y_0, Z_0) is the receiver's spatial position, and ρ_n is the observed pseudo-range between the satellite and the receiver.

The coordinate system translation matrix T from ECEF to ENU is

$$T = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin L \cos\lambda & -\sin L \sin\lambda & \cos L \\ \cos L \cos\lambda & \cos L \sin\lambda & \sin L \end{bmatrix} \tag{29.7}$$

where L and λ is receiver's geodetic latitude and longitude respectively.

Two transitional matrix A and B are introduced as

$$A = (H'H)^{-1} \tag{29.8}$$

$$B = TA_{1:3,1:3}T' \tag{29.9}$$

where $A_{1:3,1:3}$ is improved 3×3 matrix subtracted last row and column by A, then

$$GDOP = \sqrt{A_{11} + A_{22} + A_{33} + A_{44}} \tag{29.10}$$

$$PDOP = \sqrt{A_{11} + A_{22} + A_{33}} \tag{29.11}$$

$$TDOP = \sqrt{A_{44}} \tag{29.12}$$

$$HDOP = \sqrt{B_{11} + B_{22}} \tag{29.13}$$

$$VDOP = \sqrt{B_{33}} \tag{29.14}$$

29.3 Computational Results and Analysis

This paper employed BeiDou broadcast ephemeris (5GEO+5IGSO+4MEO) to compute spatial number of visible satellites and DOPs from UTC 2012-06-28 00:00:00 to UTC 2012-07-05 00:00:00. Temporal sampling was 5 min and spatial sampling was $5° \times 2.5° \times 100$ km (longitude, latitude and elevation intervals). The half-beam width angle was $\pm 10°$ for GEO and IGSO satellites and $\pm 15°$ for MEO satellites. It is noticed that the masking angle was not considered in these computations.

29.3.1 Spatial Distribution at a Certain Time

The spatial distribution for number of visible satellites and PDOP at UTC 2012-06-28 00:00:00 are shown in Figs. 29.4 and 29.5 respectively: the effective coverage areas of BeiDou regional constellation are eastern hemisphere and south and north Polar Regions, and the spatial altitude is under 1,000 km above the earth surface, these areas can meet the minimum requirements of PNT services because the number of visible satellites is more than 4 and PDOP is less than 8. The space beyond 1,000 km may not meet the minimum requirements because effective coverage areas for eastern hemisphere are decreasing, and number of visible satellites in some area for western hemisphere is less than 3. For the computation of PDOP is closely related to the number of visible satellites, the spatial distribution for these two factors are similar.

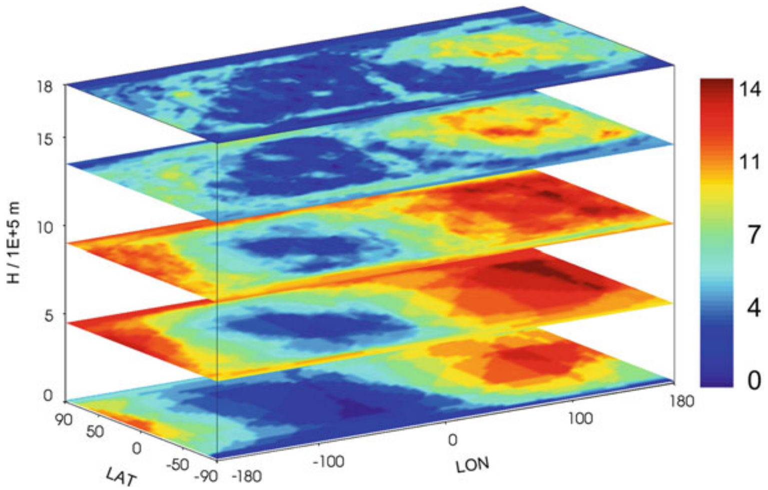

Fig. 29.4 Spatial distribution of visible satellites number at a certain time

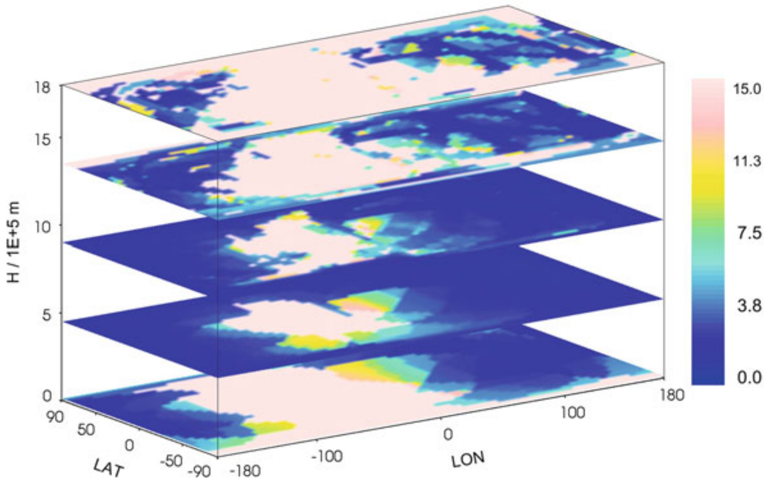

Fig. 29.5 Spatial distribution of PDOP at a certain time (High light areas were not covered by satellites)

29.3.2 *Temporal Distribution of Statistical Results*

To analyze the time-varying character of spatial coverage for BeiDou regional constellation, the number of visible satellites, worst PDOP, mean number of visible satellites, spatial effective continual time in total one week from UTC 2012-06-28 are computed respectively, and the results are shown from Figs. 29.6, 29.7, 29.8 and 29.9. Then the effective continual time segments ($N \geq 4$ and $PDOP \leq 7$)

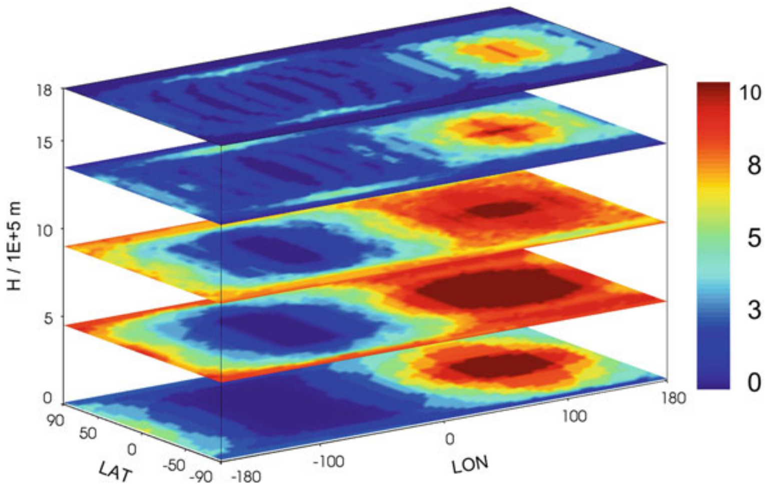

Fig. 29.6 Distribution of least visible satellites number

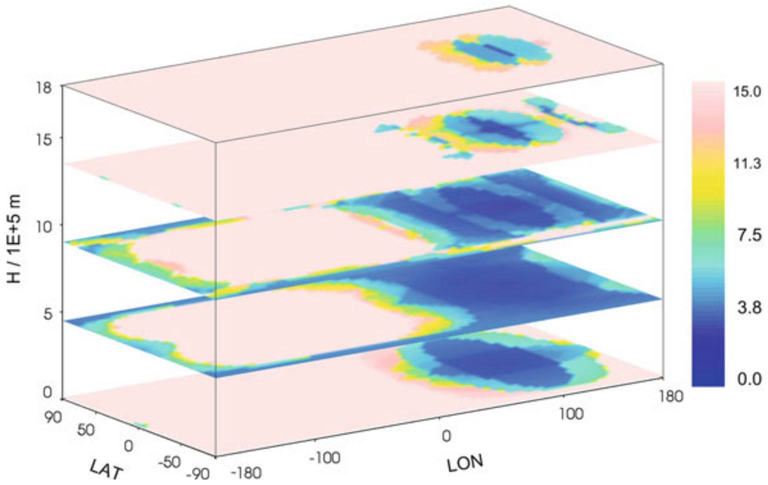

Fig. 29.7 Distribution of worst PDOP (High light areas were not covered by satellites)

are computed and analyzed at 60 spatial points which are evenly distributed. The spatial points' distribution is shown in Fig. 29.10, and the statistical results are listed in Table 29.1.

The least number of visible satellites and the worst PDOP reflects the minimum effective PNT services performance that constellation can supply to special spatio-temporal domain. As showed in Figs. 29.6 and 29.7, the space that BeiDou regional constellation can continually supply effective PNT services in one week include: eastern hemisphere and south and north Polar Regions under 1,000 km

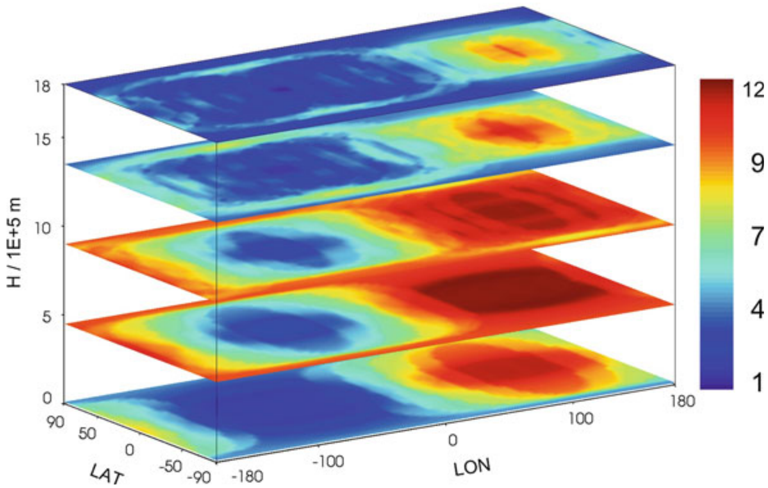

Fig. 29.8 Distribution of mean number of visible satellites

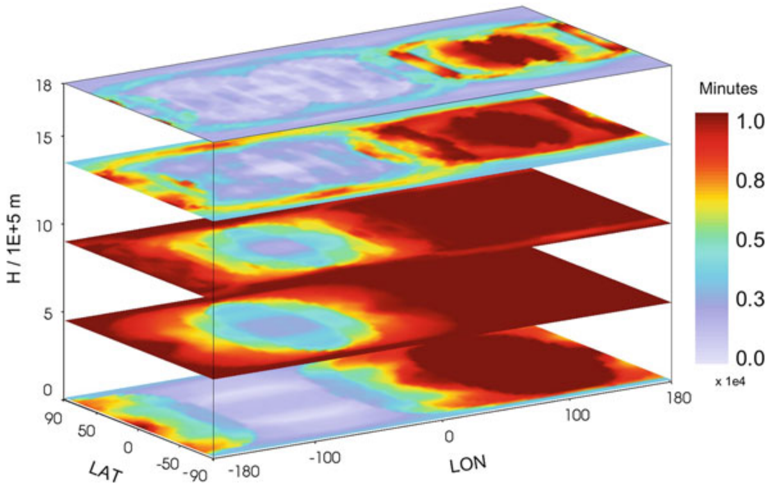

Fig. 29.9 Statistics of spatial continual time (N $\geq$ 4, PDOP $\leq$ 7)

above the earth surface. The space from 500 km to 1,000 km reach the most effective coverage, and the coverage will reduce rapidly beyond 1,000 km.

The mean number of visible satellites reflects the general distribution during certain time span, and the computed results can only for reference. As showed in Fig. 29.8, BeiDou regional constellation could provide effective PNT services for Asia–Pacific region on the ground and spatial south and north Polar Regions under 1,000 km above the earth surface.

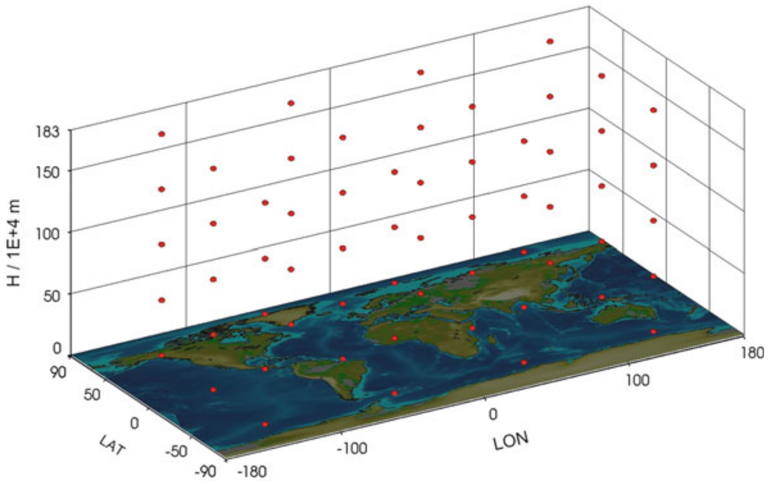

Fig. 29.10 Spatial points distribution employed for statistical results (N ≥ 4, PDOP ≤ 7)

The spatial continual time reflects the continual capability that constellation can provide effective PNT services to the spatial focused point. Figure 29.9 shows that: in the most regions of eastern hemisphere and south and north polar under 1,000 km, the continual time is no less than 1,000 min (about 6.9 days), and the regions above 1,000 km which can meet the above performance will decrease rapidly.

The statistical results in Table 29.1 show that: (1) All specifications in continual time segments statistics are symmetry in North–South direction, and this character has no correlation with elevation. (2) In the space under 1,000 km, segments in the western hemisphere are much more than the eastern hemisphere. In the spatial domain above 1,000 km, the segments in the eastern hemisphere increase rapidly with the increment of altitude, and even exceed the western hemisphere in some area. So the spaces above 1,000 km are not suitable for spatial continual navigation. (3) "Maximum continual time" performs the best in space from 500 km to 1,000 km and will decrease rapidly above 1,000 km. (4) The results of "Sum of continual time" are consistent with Fig. 29.9.

From the analysis of the above computed results of visible satellites number, PDOP, spatial distribution at a certain time, and temporal distribution, some conclusions can be summarized as follows: BeiDou regional constellation could provide effective PNT services for Asia–Pacific region mainly, for the restriction of smaller half-beam width signal to GEO and IGSO and the quantity of MEO, the spatial scope limitation is from the earth surface to an altitude of 1,000 km above the earth surface. For the higher altitude (36,000 km) of GEO and IGSO, the space from 500 km to 1,000 km above the earth surface in south and north Polar Regions

Table 29.1 Statistical results of continual time segments (N ≥ 4, PDOP ≤ 7, minutes)

Height (km)	Longitude	Latitude	Number of segments	Maximum continual time	Minimum continual time	Sum of continual time	Per centum (%)
10	−135	−60	38	575	0	4,650	46
	−45	−60	26	140	0	1,200	12
	45	−60	28	1,005	0	8,145	81
	135	−60	1	10,075	10,075	10,075	100
	−135	0	13	715	0	4,015	40
	−45	0	12	210	25	1,400	14
	45	0	7	2,080	5	9,675	96
	135	0	1	10,075	10,075	10,075	100
	−135	60	40	400	0	4,585	46
	−45	60	26	140	0	1,180	12
	45	60	29	1,000	0	8,105	80
	135	60	1	10,075	10,075	10,075	100
460	−135	−60	13	1,905	5	8,775	87
	−45	−60	32	895	0	6,480	64
	45	−60	1	10,075	10,075	10,075	100
	135	−60	1	10,075	10,075	10,075	100
	−135	0	13	1,265	30	6,930	69
	−45	0	14	325	65	2,470	25
	45	0	1	10,075	10,075	10,075	100
	135	0	1	10,075	10,075	10,075	100
	−135	60	14	1,905	5	8,785	87
	−45	60	34	895	0	6,450	64
	45	60	1	10,075	10,075	10,075	100
	135	60	1	10,075	10,075	10,075	100
910	−135	−60	35	1,340	0	8,160	81
	−45	−60	34	1,295	0	7,700	76
	45	−60	1	10,075	10,075	10,075	100
	135	−60	1	10,075	10,075	10,075	100
	−135	0	33	1,100	0	7,705	76
	−45	0	25	490	0	3,260	32
	45	0	1	10,075	10,075	10,075	100
	135	0	1	10,075	10,075	10,075	100
	−135	60	34	1,335	0	8,165	81
	−45	60	33	1,300	0	7,685	76
	45	60	1	10,075	10,075	10,075	100
	135	60	1	10,075	10,075	10,075	100

(continued)

Table 29.1 (continued)

Height (km)	Longitude	Latitude	Number of segments	Maximum continual time	Minimum continual time	Sum of continual time	Per centum (%)
1,360	−135	−60	49	500	0	4,890	49
	−45	−60	45	95	0	1,700	17
	45	−60	65	715	0	6,670	66
	135	−60	63	540	0	7,500	74
	−135	0	36	365	0	3,355	33
	−45	0	23	200	0	1,260	13
	45	0	35	625	0	8,240	82
	135	0	1	10,075	10,075	10,075	100
	−135	60	53	490	0	4,875	48
	−45	60	48	95	0	1,715	17
	45	60	65	715	0	6695	66
	135	60	63	545	0	7,545	75
1,810	−135	−60	44	290	0	3,065	30
	−45	−60	44	250	0	2,840	28
	45	−60	57	150	0	1,945	19
	135	−60	75	260	0	3,915	39
	−135	0	29	205	0	2,230	22
	−45	0	5	135	45	425	4
	45	0	36	480	5	6,580	65
	135	0	1	10,075	10,075	10,075	100
	−135	60	45	290	0	3,030	30
	−45	60	45	250	0	2,810	28
	45	60	56	180	0	1,915	19
	135	60	73	255	0	3,890	39

Note "0" in "Minimum continual time" specifies that statistical segment (meet $N \geq 4$, $PDOP \leq 7$) is just one time point

can obtain effective PNT services. Relative to eastern hemisphere, the western hemisphere where the number of effective continual time segments is lager, the sum of continual time is fewer, so it is not suitable for spatial continual and long time navigation.

29.4 Conclusions

Without considering of the effect of signal in space (SIS) power, this paper employed BeiDou regional constellation (5GEO+5IGSO+4MEO) to compute and analyze the spatial number of visible satellites and DOPs. The computational results showed that: BeiDou regional navigation satellite system could provide effective PNT services for Asia–Pacific region, south and north Polar Regions under 1,000 km, satisfied the designed service performance of BeiDou regional constellation. The computational and statistical results of the least visible satellites number, the worst PDOP, and the effective continual time segments reflected that: the space from 500 km to 1,000 km above the earth surface could acquire the best PNT services, and the coverage area is larger than the earth surface. Because of the promptly reducing of spatial coverage area, increasing of effective continual time segments and decreasing of maximum continual time and total time in all segments, the space above 1,000 km could not acquire effective PNT services.

This paper just discussed two main factors that are number of visible satellites and PDOP in spatial coverage performance evaluation. In further research, another factor that is signal power should be taken into consideration. Moreover, the impact of spatio-temporal resolutions on the computing results of spatial coverage performance deserves further study.

Acknowledgments The authors wish to acknowledge the support of Wuhan University for providing BeiDou broadcast ephemeris in developing this paper.

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